Review Problems

March 22, 2017

1. (Fall 2006, Exam 3, #1) Which of the following series converge?

   (a) $S_1 = \sum_{n=1}^{\infty} \frac{n^3 + 9n^2}{300n^4 + 3n}$

   (b) $S_2 = \sum_{n=1}^{\infty} \frac{8n^2 + 7n}{n^4 + 9n^3}$

   (c) $S_3 = \sum_{n=1}^{\infty} \frac{8n^6 + 7n}{600n^5 + 200n^3}$

2. (Fall 2006, Exam 3, #2) Which of the following is true about the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n^2}$?

   I) It converges by the integral test

   II) It converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

   III) It diverges by the comparison test with $\sum_{n=1}^{\infty}$

3. (Fall 2006, Exam 3, #3) Let $f(x)$ be a function defined for $x \geq 1$, such that $0 \leq f(x) \leq 1$ for all $x \geq 1$. What can be said about the convergence of the series
$$S_1 = \sum_{n=1}^{\infty} \frac{f(n)}{n}, S_2 = \sum_{n=1}^{\infty} \frac{f(n)}{n^2}?$$

4. (Fall 2006, Exam 3, #4) Using that $\lim_{x \to 0} \frac{\sin x}{x} = 1$, and the limit comparison theorem, what can be said about the convergence of the series
$$S_1 = \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right), S_2 = \sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)?$$

5. (Fall 2007, Exam 3, #4) Suppose we want to approximate the sum of the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3}$ by the sum of $s_m = \sum_{n=1}^{m} (-1)^n \frac{1}{n^3}$ of the first m terms. By the theory of alternating series, the error will be less than 10^-3 provided $m =$ what?

6. (Fall 2006, Exam 3, #5) Find the smallest number of terms which one needs to add to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 n!}$ with an error strictly less than 10^{-3}.